Indian J. Phys. 65A (3), 255-257 (1991)

Nucleon-Nucleon spacing distribution in a few nucleon system

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Received 10 October 1990, accepted 18 March 1991

Few nucleon systems like deutron and Helium have been the subject of study for quite some time. In most of these studies one is either interested in the ground state energy and the static electromagnetic moments of these systems or the two-body correlations. The object of the present work is to add a new component in the study of a few nucleon system by studying the spacing of nucleons in a light nucleus.

The concept of spacing was first introduced by Wigner (1957) when he developed his theory of the distribution of eigenvalues of a Hamiltonian matrix. The precise definition of the spacing given by him was that it is an interval of energy devoid of any eigenvalues while exactly one eigenvalue lies at the end of the given interval and the remaining ones lying anywhere outside this interval. In further development it was found that the eigenvalues of the Hermitian Hamiltonian behave like a system of one dimensional fermions. Our objective here is to use these concepts in three dimensions for a real system of nucleons and derive an expression for nucleon-nucleon spacing distribution in a few nucleon system. It should be emphasized here that the concept of spacing is quite different than the study of two body correlations (1962) where any number of nucleons may lie between the two nucleons.

In order to keep the problem of nucleon-nucleon spacing simple we shall in the present formulation use shell-model picture only. Let us consider a three nucleon problem in which all the three nucleons are moving in the 1s state of harmonic oscillator. Let $P(\{\bar{r}_i\})$, $i=1, 2, 3$ denote the joint spatial probability density of three nucleons. For the 1s state it is given by

$$P(\{\bar{r}_i\}) \prod_{i=1}^{3} d\bar{r}_i = K\left[\exp\left(-\sum_{i=1}^{3} r_i^2\right)\right] \prod_{i=1}^{3} d\bar{r}_i, \tag{1}$$

where the appropriate antisymmetric part of three nucleon wave function have some given spin and iso-spin is integrated out. K is the normalization constant

$$K = [P(\{\bar{r}_i\}) d\bar{r}_i]^{-1} \tag{2}$$

For simplicity the exponent in eq. (1) is taken to be unity.

The analogue of one dimensional spacing will now be that if one nucleon is at point $\bar{r}_1$ and another at point $\bar{r}_2$, then within a sphere of radius $|\bar{r}_1-\bar{r}_2|$ there are no nucleons, all the other remaining $(N-2)$ nucleons could be anywhere outside this sphere. Denoting $\bar{r}_1-\bar{r}_2$ by $\bar{r}$ we can write this probability using eq. (1) as

$$P(\bar{r},\bar{r}_1)d\bar{r}d\bar{r}_1=k\exp\left(-r^2-2r_1^2-2\bar{r}.\bar{r}_1\right)$$
$$\left[1-\pi^{-3/2}\int\left[\exp\left(-r_3^2\right)d\bar{r}_3\right]d\bar{r}d\bar{r}_1,\right. \tag{3}$$

where the integration in the last integral is carried out over the region $|\bar{r}_3-\bar{r}_1|^2 \leqslant r^2$ and k is the new normalization constant. We now let the nucleon 1 lie anywhere in space and average over the directions of the vector $\bar{r}$. This will give the probability that if one finds a nucleon anywhere in space, then there will be no other nucleons in the sphere of radius r around it. The second nucleon will be anywhere on this sphere while the third one will be any where outside this sphere. This probability density $P(r)$ using eq. (3) is found to be

$$P(r)=\left[\frac{8}{\pi}r\exp\left(-r^2\right)\right]\left[\frac{1}{2}\sqrt{\frac{\pi}{2}}\,r\exp\left(\frac{r^2}{2}\right)\right.$$
$$+\frac{\sqrt{3}}{4}\left(\exp\left(\frac{r^2}{3}\right)-\exp\left(-r^2\right)\right)-\frac{1}{4}\sqrt{\frac{\pi}{2}}\,r\exp\left(\frac{r^2}{2}\right)$$
$$\left.\left(erf\left(\sqrt{\frac{3}{2}}r\right)\right)\right], \tag{4}$$

where erf denotes error function Abramowitz M and Stegun I A 1965. $P(r)$ is normalized such that

$$\int_0^\infty P(r)dr=1. \tag{5}$$

We note from eq. (4) that nucleon-nucleon spacing distribution vanishes both when $r\to 0$ and $r\to\infty$. In this way it has close resemblance with the well-known Wigner's spacing distribution.

Even though the expression for $P(r)$ given by eq. (4) is quite involved, it is fairly straight-forward to calculate one of its most important characteristic, namely the ratio of mean-square deviation to the square of the mean. It is given by

$$\frac{\langle r^2\rangle-\langle r\rangle^2}{\langle r\rangle^2}=0.2. \tag{6}$$

We shall now pass a few concluding remarks.

We have shown how to extend the concept of spacing introduced by Wigner to a system of few nucleons. An explicit expression for nucleon-nucleon spacing distribution $P(r)$ is derived for a system of three nucleons all moving in 1s state.

From eq. (6) we find that the dispersion of spacing around the mean is quite small which means most of the time nucleons are separated by the average distance $< r >$.

It should further be remarked that the ratio 0.2 given by eq. (6) is quite close to the value which one obtains for the corresponding ratio for the spacing distribution given by Wigner. At present it is hard to say if it has any deeper significance for the system of few nucleons.

Lastly we remark that at present there seem to be no experimental data available which could be used to get information about $P(r)$ but it is hoped that such a data may be available in future and it will be interesting to see how the present predictions about the behaviour of $P(r)$ given by eqs. (4) and (6) compare with experiment.

References

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